Basic Rectifier Relationships and Design Considerations for use with Rectennas Part 1

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Diode Properties

Rectifier diodes are characterized by having an exponential relationship between current through the diode and voltage across the diode:

$$Id = e^{b(Vf - Vth)},$$

where Id is the diode current, Vf is the voltage across the diode, Vth is the threshold voltage at which current begins to flow, and b is a constant determined by the materials and construction of the diode. Below the threshold voltage, very little current flows while above the threshold voltage the current increases rapidly and approaches a linear resistive characteristic.

This is often approximated as two regions with the transition between the regions at a voltage near the threshold voltage. With this approximation, the voltage drop across a diode in the conduction region may be expressed as:

$$Vf = Vth + Rd * Id \text{ (when Vf is greater than Vth.)} \qquad (1)$$

This may be rearranged as:

$$Id = (Vf - Vth)/Rd \text{ (when } Vf > Vth) \qquad (2)$$

and

$$Id = 0 \text{ (when } Vf <= Vth).$$

Power loss in the diode is the product of the voltage across the diode times the current through the diode, so from equation 1 the power loss is:

$$Pd = Vf * Id$$

$$Pd = Vth * Id + Rd * Id^2 \qquad (3)$$

Vth for a particular diode is determined mostly by the materials used to make the diode. Rd is determined partly by the materials used to make the diode and partly by the cross-sectional area of the diode junction.

In the reverse bias region where the diode blocks current flow, there will be a junction capacitance Cj that is determined mostly by the cross-sectional area of the junction. There will also be a reverse voltage at which the diode begins conducting in the wrong direction called the breakdown voltage (Vbr). Vbr is determined by the materials and doping profile used to make the diode. Applying a reverse voltage in excess of Vbr is likely to cause damage to the diode.

Desirable characteristics for a rectifier are low reverse-bias junction capacitance (Cj), low forward threshold voltage (Vth), low dynamic resistance (Rd), high reverse breakdown voltage (Vbr), high heat-dissipating ability and low lead inductance and holder capacitance introduced by the device package. Unfortunately when steps are taken to optimize one of the above, the others generally degrade and a compromise must be made.

There are two semiconductor diode technologies that may be considered for this application – silicon schottky (Si) and gallium arsenide (GaAs). Silicon is characterized by low forward threshold voltage at the cost of higher dynamic resistance. Typical characteristics for a silicon diode with junction capacitance of 1 or 2 pF are forward threshold voltage of 0.3 volts, dynamic resistance of 10 ohms and reverse breakdown voltage of 15 volts. Typical characteristics of a gallium arsenide diode with junction capacitance of 1 or 2 pF are forward threshold voltage of 0.7 volts, dynamic resistance of 1.5 ohms and reverse breakdown voltage of 60 to 70 volts. The cross-sectional area of the semiconductor junction can be increased to reduce dynamic resistance but this increases junction capacitance and junction capacitance much above 2 pF causes too much of the radio frequency energy to be shunted around the diode resulting in low efficiency. Forward threshold voltage and dynamic resistance can be decreased by increasing the impurity doping levels but this reduces the reverse breakdown voltage, limiting power handling capability.

Rectifier Circuit Design

With a diode of any given characteristics, the rectifier operating efficiency will be greatly influenced by the circuit surrounding the diode. In the following discussion, four basic theorems are of great importance:

1. **Maximum Power Transfer Theorem** – When a signal is obtained from a source having a source impedance of Rs, maximum power will be transferred to the load when the load impedance is the complex conjugate of the source impedance. In the case where the source impedance is purely resistive, this simplifies to requiring that the load impedance be equal to the source impedance. Figure 1 illustrates this case using a Spice simulator with a signal source of 2.45 GHz with source and load resistances of 50 ohms and with amplitude chosen to develop 150 milliwatts in the load:

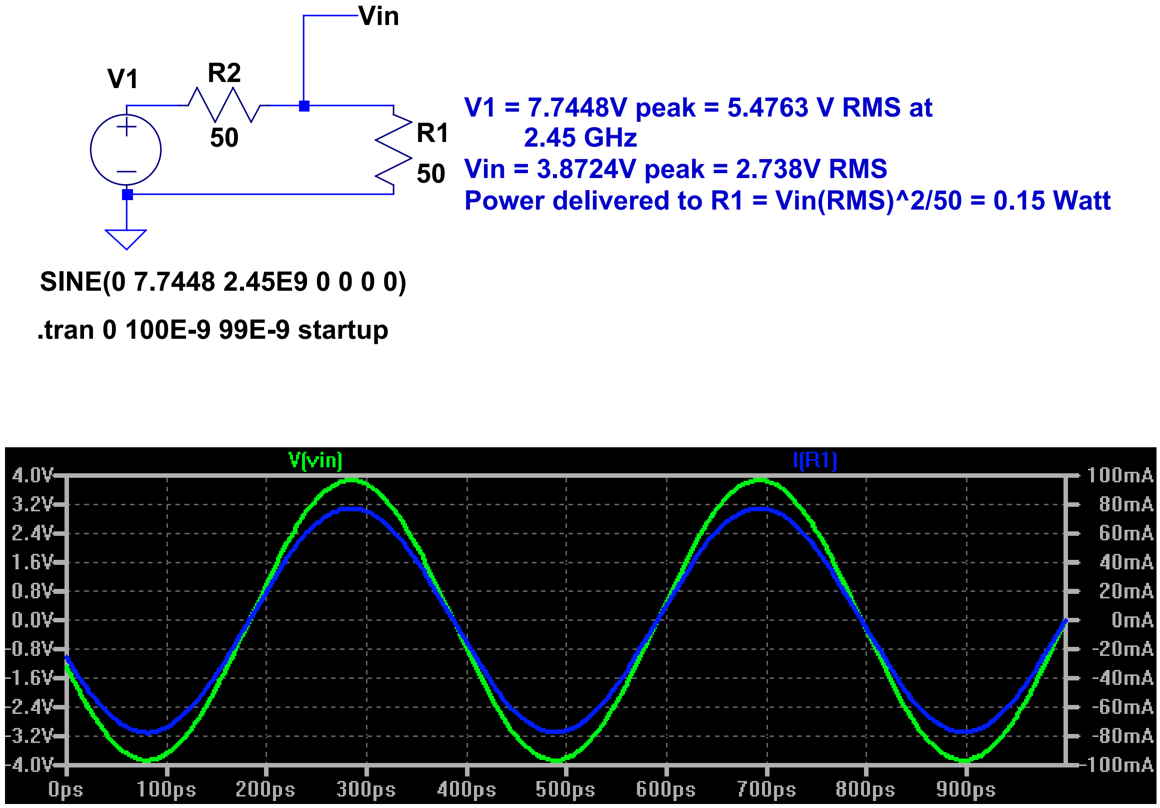


Figure 1
Relationships for Maximum Power Transfer

Note that the voltage and current waveforms are sinusoidal in form, and the peaks of the two waveforms are aligned along the horizontal axis (The voltage and current are "in phase"). A "pure" signal (voltage or current) containing energy at a single frequency will always be sinusoidal in form, and a requirement for maximum power transfer is for the voltage and current to be in phase. 50 ohm resistances are chosen because most antennas are designed to present this value of source impedance, and 150 milliwatts is chosen as a power level representative of the power expected from the receiving antenna in the power beaming demo system.

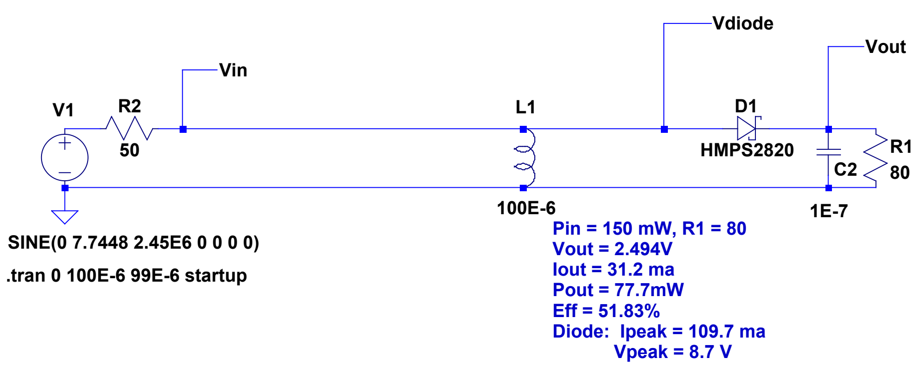
2. **Fourier Series Theorem** – This theorem states that any periodic waveform of frequency F can be represented as the sum of sinusoidal waveforms at frequencies that are integer multiples of F. A sinusoidal waveform only contains energy at F (called the "fundamental"), while any other periodic waveform contains energy at other multiples (called "harmonics") of F.

3. Reactive components (inductors and capacitors) do not "use up" energy, but instead store energy during one part of a cycle and return it back to the circuit during another

part. Inductive and capacitive reactances are opposite in nature, and one may be used to partially or totally cancel the other. In a circuit where the cancellation is complete, a condition called "resonance" is produced. These properties are useful in making filtering and impedance-transforming circuits.

4. Non-linear devices (such as diodes) will produce a result where the voltage and current are not identical in form. Such a result is called "distortion" and the Fourier Series Theorem indicates the resultant waveform will contain harmonics. A non-linear load cannot produce maximum power transfer since the voltage and current waveforms are not identical in form. (In other words, the source impedance is a pure resistance while the load is not a pure resistance, and thus the load impedance is not equal to the source impedance.)

The manner in which the above relationships are important is illustrated in Figure 2. This circuit uses an Avago Technologies HMPS 2820 silicon schottky diode with a threshold voltage of about 0.3 volts, a dynamic resistance of 10 ohms, and a junction capacitance of 1 pF. The RF source (V1 and R2) is chosen to represent a 50 ohm source that will deliver 150 milliwatts into a 50 ohm load as in Figure 1. L1 is included to provide a DC return path for the diode with high enough inductive reactance to have little effect on the RF signal. C2 provides filtering of the pulsating signal produced by the diode to obtain smooth DC output. The output load resistance (R1) was adjusted to produce highest power output (Volts x Amps) in R1. For this example, the frequency was set to 2.45 MHz (1000 times lower than our intended frequency) to make the effect of diode junction capacitance trivial. (We will deal with this later.)



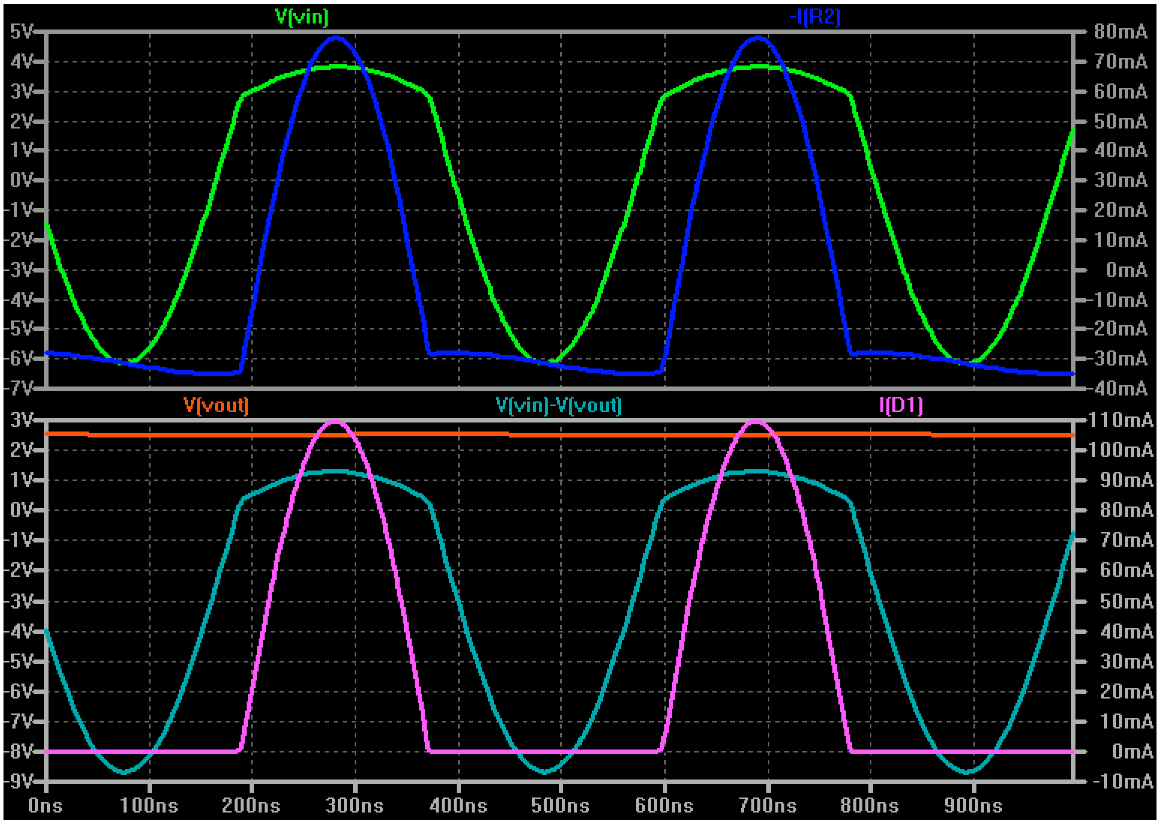


Figure 2
Low Frequency Simple Rectifier Circuit

The upper graph of the input voltage and current shows that the voltage and current waveforms are highly distorted with the voltage being “flattened” on top and the current flowing only during the small part of each cycle when the source voltage is greater than the output voltage. The result is low efficiency (51.83%) and in addition the harmonic energy produced by the diode will flow back into the antenna and be re-radiated. This harmonic energy is dominated by the 2nd harmonic for the type of waveforms in Figure 2. The largest contributor to poor efficiency is failure to meet the requirements of the maximum power transfer theorem, but an additional contribution comes from the high peak current (109.7 ma) in the diode. This high peak current occurs because current only flows during a small portion of the cycle (Low “conduction angle”) and in order to produce any given average current, the current during the time current flows must be higher than the average. This condition is also called a “high peak-to-average ratio”. (Peak to average ratio in Figure 2 is 3.52.) Equation 3 from above shows that power loss in the diode increases rapidly with current because of the Id^2 term so it is desirable to keep the peak to average ratio as low as possible.

If the source can’t “see” the diode harmonics, then the waveforms at the Vin node will be sinusoidal and it will be possible to meet the requirements of the maximum power transfer theorem at the antenna input. This suggests addition of a low pass filter that will pass the fundamental and block the harmonics. While this is the basic approach that will be taken, it is difficult to obtain high attenuation of the 2nd harmonic with a simple filter. A parallel-resonant circuit ("trap") at the 2nd harmonic in series with the diode (C4 and L2) will also be used to block 2nd harmonic energy as shown in Figure 3:

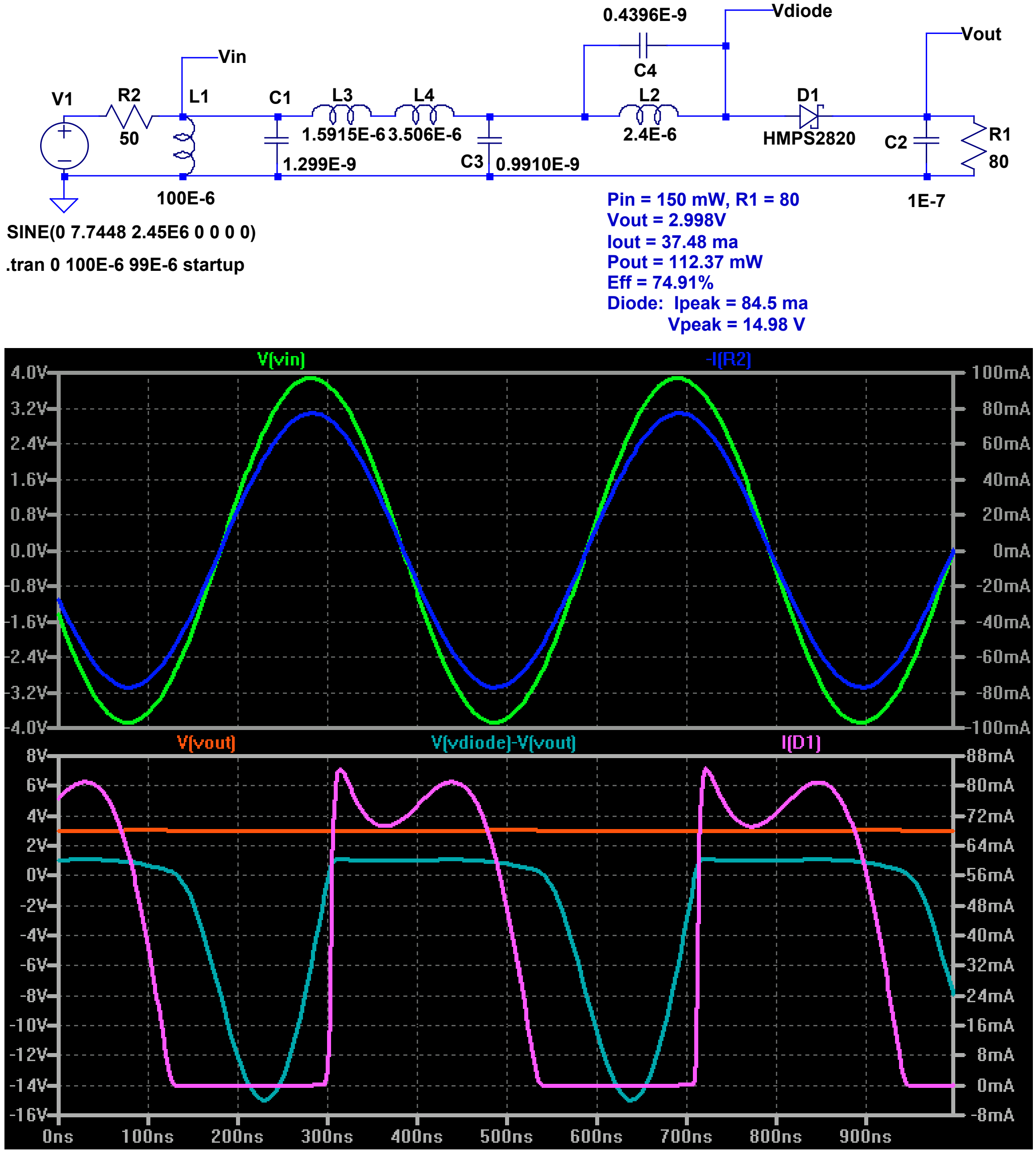
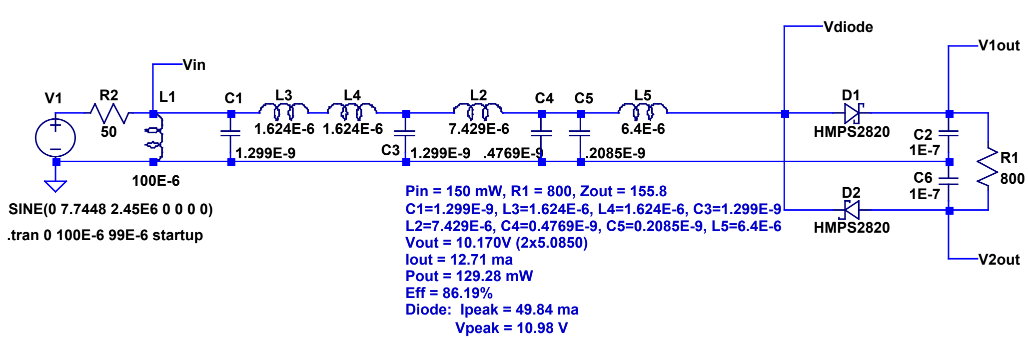


Figure 3
Low Frequency Rectifier with Low Pass Filter and 2nd Harmonic Trap

The following points should be noted from this figure:

- The benefits of the low pass filter (C1, L3, L4 and C3) and 2nd harmonic trap (C4 and L2) are as expected. The input voltage and current are very nearly sinusoidal (low harmonic content) and the voltage and current were adjusted to be in phase by varying the L/C ratio of the 2nd harmonic filter. The voltage and current at the input were adjusted for a good impedance match by varying the impedance transformation of the filter. The peak voltage and current at the input are nearly identical to Figure 1, indicating a good match between source and load impedance. Overall efficiency is 74.91% in comparison with 51.83% in the circuit of Figure 2.
- Peak diode current is 84.5 ma to produce an output current of 37.48 ma (Peak to average ratio of 2.25). This is much better than the peak-to-average ratio of 3.52 in Figure 2.
- Peak reverse voltage on the diode is 14.98 volts. This is getting dangerously close to the maximum diode reverse voltage rating of 15 volts.
- Load impedance was left at the same value (80 ohms) used in Figure 2 so the difference in performance between the two circuits is solely due to the filters.
- Efficiency would probably improve by increasing the load impedance, but this will result in even higher reverse voltage across the diode.
- The low pass filter (C1, L3, L4 and C1) also provides impedance transformation from a 50 ohm input impedance to a 141.5 ohm output impedance. This transformation was adjusted to produce a 50 ohm input impedance to the low pass filter.

The two problems noted in Figure 3 (High peak current in the diode and high reverse voltage across the diode) can both be improved by use of a full-wave rectifier. A full wave rectifier will inherently produce low 2nd harmonic levels because the 2nd harmonic produced by one diode is cancelled by the equal but opposite 2nd harmonic produced by the other. (Assuming good matching between diodes.) The predominant harmonic will be the 3rd and a practical low pass filter will provide good attenuation of the 3rd and higher harmonics so the 2nd harmonic "trap" of Figure 3 is omitted. These considerations result in the circuit of Figure 4:



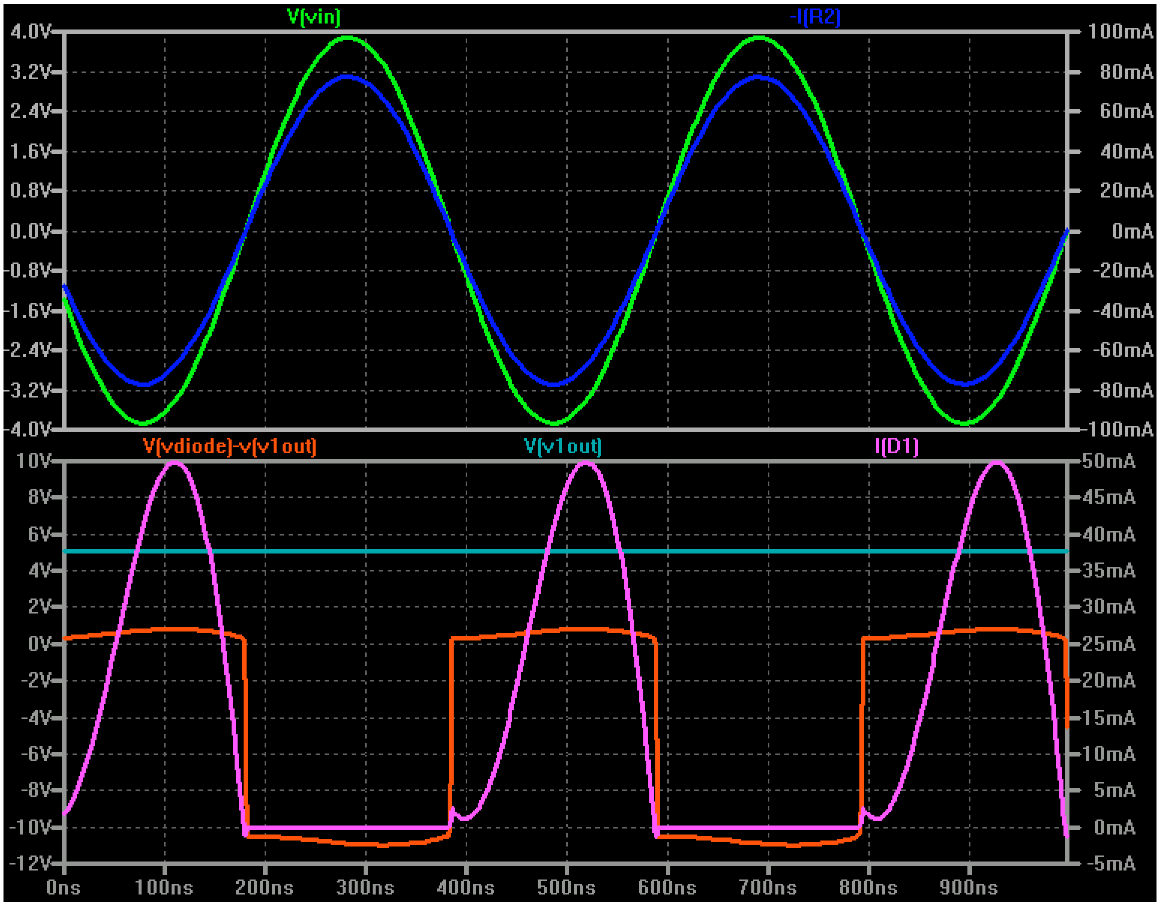


Figure 4
Low Frequency Full Wave Rectifier with PI-T low pass filter

In developing this circuit, several combinations of filter topology and impedance transformation were tried. The first filter section (C1, L3, L4, and C3) produced little change in performance with either PI or T filter topologies and there was also little change in performance as the intermediate impedance at the junction of the first and second filter sections was varied. Conversely, a T filter topology for the second filter section produced much better performance than a PI topology. Either filter topology may be viewed as a cascaded pair of L networks where the first L section transforms to an intermediate impedance and the second L section transforms from this intermediate impedance to the output impedance. For the PI topology, the intermediate impedance is less than both the input and output impedances, while for the T topology, the intermediate impedance is higher than both the input and output impedances. (The intermediate impedance for the second filter in Figure 4 is 311.6 ohms, the input impedance is 50 ohms and the output impedance is 155.8 ohms.) While not a general proof, this relationship suggests that the optimum driving impedance for the rectifier diode is a high source impedance at the fundamental and all harmonics. Note: The value of L5 = 6.4E-6 henries was adjusted down from the value calculated for the T filter of L5 = 10.12E-6 to bring the input current I(R2) in-phase with the input voltage V(vin). This is necessary to correct for the effects of the diode non-linearity.

This circuit was evaluated with several values of load resistance (R1). Best efficiency was obtained with a load of 800 ohms and the schematic and waveforms are shown for this case. While efficiency decreases for other values of R1, efficiency remained good for loads from 400 to 1000 ohms showing that the choice of load resistance is not highly critical. This is fortunate for the power beaming demo system since the load will be LED's and the only choices are how many LED's to connect in series. Typical LED's have a forward voltage around 2.2 volts and these results suggest that the load should be four LED's in series for a total voltage of 8.8 volts. This will produce a load impedance of about 613 ohms in Figure 4 and the rectifier would be expected to deliver about 14.35 ma into the LED's at an efficiency of about 84.2% (total output power of 126.24 mw). Some small improvement in efficiency can probably be obtained by re-optimizing the circuit for the 613 ohm load and by increasing the value of the intermediate impedance of the second filter section, but this would be of mostly academic interest since the diode junction capacitance will prohibit this approach when the design is moved from 2.45 MHz to 2.45 GHz.

Author's Note: Manual optimization of these circuits using the Spice simulator is a very tedious process. Adjustment of one component generally requires compensating adjustments of others to obtain best performance. (This is illustrated in Figure 4, where changing R1 required adjustment of L2, C4, C5 and L5 to obtain correct impedance matching and phase alignment of the input voltage and current.) The efficiencies shown in Figures 3 and 4 are the best results obtained by the author but are not claimed to be global optimums.

Even though the power level needed for the table top demo system may be a little above the transition point where a GaAs diode would provide better performance, the author believes that there will not be a large penalty in using a silicon diode. Silicon diodes are much more readily available than gallium arsenide and are recommended for the demo system. A suitable silicon diode is the Avago Technologies HMPS 2820 family used in the circuits of Figures 2, 3 and 4.

The next step in evolving a finished design is to translate the low frequency implementation in the above to 2.45 GHz. This process will show the importance of the diode junction capacitance at high frequencies and will develop a method of dealing with this issue. This work will be continued in a follow-on Part 2 of this study in a separate document.

In a study conducted for NASA by William Brown at Raytheon, a rectification efficiency of 85% was obtained at a power level of 1.5 watts with a load resistance of 400 ohms in a half-wave rectifier circuit at 2.45 GHz. This study used a custom-made GaAs diode in a circuit with the diode in a shunt rectifier configuration with a resonant output circuit. (See http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19870010123_1987010123.pdf) Brown's work and the results herein establish a goal for the lower-power version we are developing using silicon diodes. The online reference http://www.sspi.gatech.edu/wptshinohara.pdf also contains many useful references for those interested in additional study.